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# Absolute structure determination using *CRYSTALS*

Richard Ian Cooper,<sup>a\*</sup> David John Watkin<sup>a</sup> and Howard D. Flack<sup>b</sup>

<sup>a</sup>Chemical Crystallography Laboratory, University of Oxford, 12 Mansfield Road, Oxford, Oxfordshire OX1 3TA, England, and <sup>b</sup>Chimie minérale, analytique et appliquée, University of Geneva, Geneva, Switzerland. \*Correspondence e-mail: richard.cooper@chem.ox.ac.uk

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A study of post-refinement absolute structure determination using previously published data was carried out using the *CRYSTALS* software package. We show that absolute structure determination may be carried out optimally using the analyses available in *CRYSTALS*, and that it is not necessary to have the separate procedures *absolute structure determination* and *no interest in absolute structure* as proposed by Flack [*Chimia* (2014), **68**, 26–30].

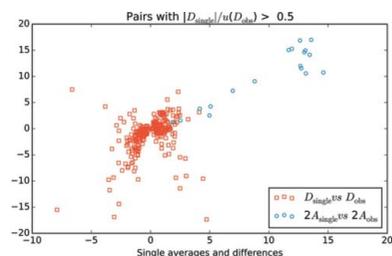
## 1. Introduction

Three elements have combined to make this study of absolute structure determination feasible and significant.

The first element is the *CRYSTALS* software package (Betteridge *et al.*, 2003). This study relied on its flexible least-squares refinement procedures associated with its capacity for optimizing and analysing weights. Also of great importance is the capacity of *CRYSTALS* to present data analysis in visual form. The normal probability plots, the plots of  $D_{\text{obs}}$  against  $D_{\text{single}}$ , the histograms of observed values of the Flack (1983)  $x$  parameter and the plots for evaluating data coverage and completeness were of particular value. Version 14/Build 5792 of *CRYSTALS* was used for these analyses; a Windows version is available for download from <http://www.xtl.ox.ac.uk/tag/crystals-release.html>.

The second element is a list and analysis of a very varied set of crystal structure determinations of noncentrosymmetric crystal structures (Flack, 2013). All determinations of noncentrosymmetric crystal structures published in *Acta Crystallographica Section C* in the years 2011 and 2012 were analysed in a recent publication (Flack, 2013). For those structures for which the deposited X-ray diffraction intensity data were suitable, the intensity data were separated into the following three classes: paired ( $hkl$  and  $\bar{h}\bar{k}\bar{l}$ ) acentric reflections, unpaired acentric reflections and centric reflections. Moreover plots of  $A_{\text{obs}}$  against  $A_{\text{model}}$  and  $D_{\text{obs}}$  against  $D_{\text{model}}$  had been prepared.  $A(hkl)$  and  $D(hkl)$  are, respectively, the average intensity of Friedel opposites  $hkl$  and  $\bar{h}\bar{k}\bar{l}$ , i.e.  $A = \frac{1}{2}[|F(hkl)|^2 + |F(\bar{h}\bar{k}\bar{l})|^2]$ , and their difference, i.e.  $D = |F(hkl)|^2 - |F(\bar{h}\bar{k}\bar{l})|^2$ . Flack (2013) also provided a comparative table of values drawn from these publications which are identified by their *Acta Crystallographica Section C* co-editor code in the following. This table of values is available at <http://dx.doi.org/10.1107/S0108270113014789/ln3158sup1.pdf>.

The third element is a complete set of recommendations for best procedures for absolute structure determination (Flack, 2014). These recommendations are based on the following works: Bernardinelli & Flack (1985), Le Page *et al.* (1990), Flack & Bernardinelli (1999, 2000, 2008), Flack (2008, 2013,



2014), Hooft *et al.* (2008, 2010), Flack *et al.* (2011), Thompson & Watkin (2011) and Parsons *et al.* (2012, 2013).

This study is thus an experimental test of the procedures of Flack (2014) using *CRYSTALS* (Betteridge *et al.*, 2003) on selected data sets from Flack (2013). In this way, it has been possible both to make the rather idealized recommendations of Flack (2014) of practical value for the structure analyst, and to ensure that the procedures implemented in *CRYSTALS* follow these recommendations. As will be explained in this paper, some minor modifications to the recommendations proved to be necessary and some minor bugs in *CRYSTALS* were corrected. Furthermore, it has been possible to find an excellent procedure for analysing the effect of selection and weighting of intensity data used in the post-refinement determination of the Flack (1983) parameter, using tools that have been available in *CRYSTALS* for several years.

## 2. Experimental

### 2.1. Obtaining unbiased atomic parameters

The first step in the Flack (2014) recommendations seeks atomic parameters (positional coordinates and atomic displacement parameters) unbiased by the effect of resonant scattering. It was first considered that the safest way to achieve this result would be to undertake least-squares refinement on data containing both  $|F_{\text{obs}}|^2$  of the centric reflections and  $A_{\text{obs}}$  of the paired acentric reflections. The data of unpaired acentric reflections are not used in such a procedure. Such data correspond to a crystal twinned by inversion with the two domains present as 50% of the crystal, modelled exactly by a Flack (1983) parameter of  $x = 0.5$ . So, in forming the values of  $A_{\text{obs}}$ , an unweighted average of  $|F_{\text{obs}}(hkl)|^2$  and  $|F_{\text{obs}}(\bar{h}\bar{k}\bar{l})|^2$  is taken in order to have intensity data corresponding to a well-defined physical state. The standard options of *CRYSTALS* do not allow such a refinement. However, from the spreadsheet of each structure made for Flack (2013), it was possible to generate a file containing the required data which was read into *CRYSTALS*. The structure determinations fg3257, qs3016, bm3104III and eg3071I were treated in this way

(Table 1). Least-squares refinements were carried out on each of these data sets, fixing the Flack (1983) parameter at a value of 0.5. Results very similar to those published by the original authors were obtained.

The treatment of the data of ov3013I revealed the limitations of this technique. Least-squares refinement on data as generated above was highly unstable. The cause of this instability was easy to detect. The published data set has 481 paired acentric, 1778 unpaired acentric and three centric reflections for 343 variable parameters. Consequently, the ratio of the number of data to the number of parameters is approximately 1.4 [*i.e.*  $(481 + 3)/343$ ]. This condition was circumvented by undertaking the least-squares refinement on all data [*i.e.*  $(2 \times 481) + 1778 + 3$  reflections] and allowing the Flack (1983) parameter to vary from a starting value of 0.5. The refinement became stable and yielded results very similar to those of the authors of ov3013I. The purpose of the variable Flack (1983) parameter is not to determine its value but to ensure that the atomic parameters are unbiased by the effect of resonant scattering. This technique is one of the standard options of *CRYSTALS* and was used subsequently in the remaining structures which we studied, *i.e.* wq3001, wq3017, fn3089I, fg3255III and ku3043 (Table 1). Reanalysis of fg3257, qs3016, bm3104III and eg3071I using this method resulted in no significant changes in the value of the Flack parameter or its standard uncertainty.

### 2.2. Determination of the Flack parameter

Following the recommendations of Flack (2014), one determines the value of the Flack (1983) parameter by making a straight-line fit passing through the origin, to  $D_{\text{obs}}$  against  $D_{\text{single}}$ . This is often called a post-refinement determination of the Flack (1983) parameter and could be carried out directly in *CRYSTALS* without the need for any modification of the software. The value of the Flack (1983) parameter calculated according to the techniques described here is labelled 'Post refinement Flack' in the output of *CRYSTALS*. The appropriate  $D_{\text{obs}}$  values are obtained from the paired acentric reflections, implying that the unpaired acentric and the centric

**Table 1**  
Values of the Flack (1983) parameter.

Each structure is identified by its *Acta Crystallographica Section C* co-editor code. 'Published' indicates that the value of the Flack (1983) parameter was taken from the published paper and 'CRYSTALS' indicates that it was obtained according to the method described in §2.2. 'Filter' gives the chosen value of  $z$  in  $|D_{\text{single}}|/u(D_{\text{obs}}) > z$ .

Structure	Friedel <sub>stat</sub>	Flack (1983) parameter, $x$		Filter, $z$	Friedel pairs		Reference
		Published	CRYSTALS		Total	Used	
bm3104III	22	−0.07 (18)	−0.01 (4)	0.5	1221	284	Frampton <i>et al.</i> (2011)
eg3071I	101	0.04 (3)	0.048 (16)	0.5	3637	513	Wölper <i>et al.</i> (2011)
fg3255III	113	0.59 (6)	0.58 (2)	0.5	2559	399	Ojala <i>et al.</i> (2012)
fg3257	279	0.028 (11)	0.028 (3)	0.5	3265	2764	Bojarska <i>et al.</i> (2012)
fn3089I	389	0.36 (5)	0.38 (2)	0.5	1367	425	Hendsbee <i>et al.</i> (2011)
ku3043	1386	0.50 (6)	0.500 (18)	0.5	156	95	Cora <i>et al.</i> (2011)
ov3013I	3	6 (3)	5 (17)	0.0001	482	399	Li <i>et al.</i> (2012)
qs3016	55	0.20 (6)	0.20 (3)	0.5	2107	310	Ślepokura (2012)
wq3001	558	0.00 (2)	0.02 (2)	2.0	2089	397	Kefi <i>et al.</i> (2011)
wq3017	467	0.402 (14)	0.411 (6)	0.5	2530	1342	Zhong & Qian (2012)

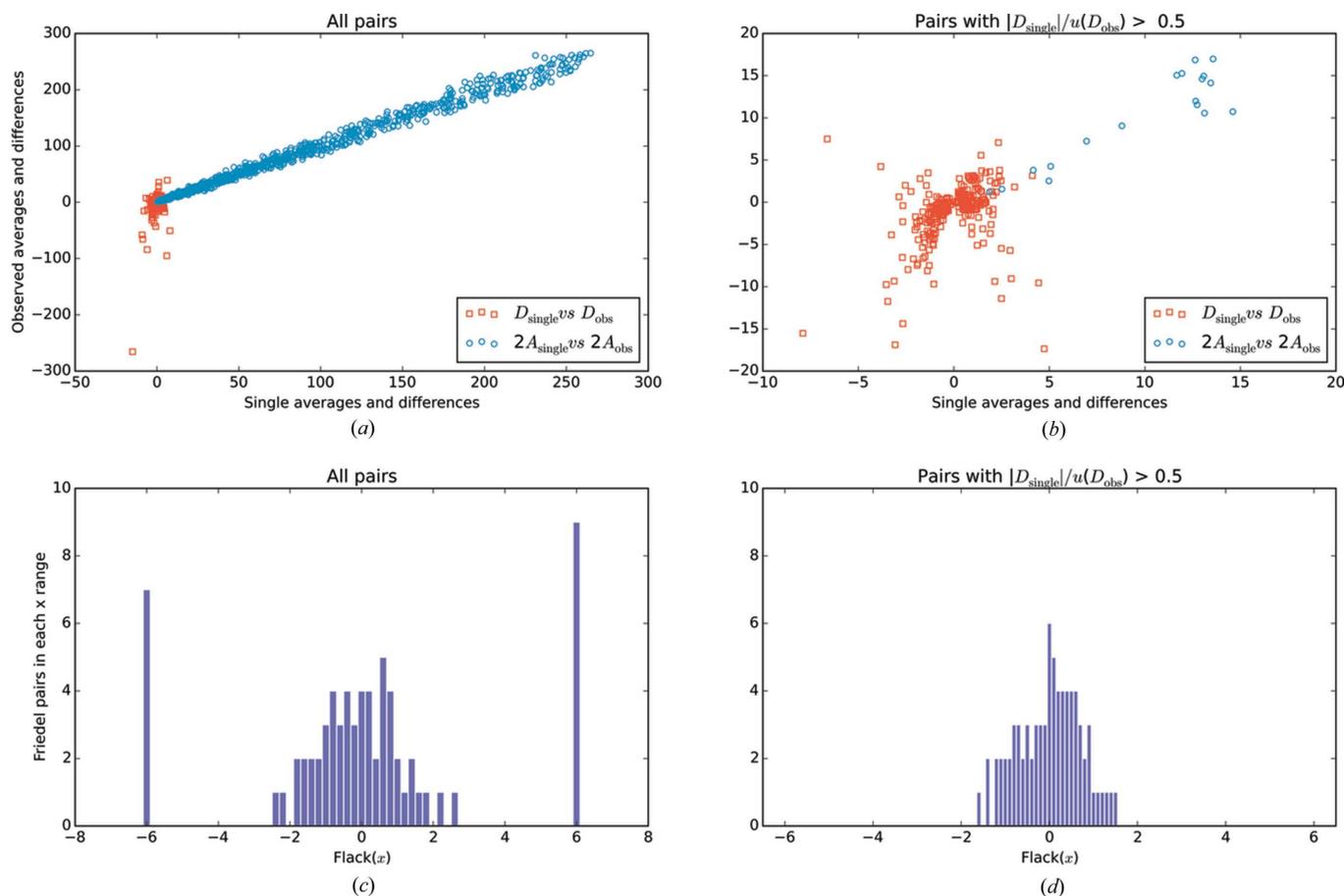


Figure 1

Plots from bm3104III (Table 1), with  $x = -0.01$  (4), showing (a)  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data (open red squares), with  $2A_{\text{obs}}$  against  $2A_{\text{single}}$  (open blue circles) included to show the quality of the fit of the data from the least-squares analysis described in §2.1; (b)  $D_{\text{obs}}$  against  $D_{\text{single}}$  (open red squares) and  $2A_{\text{obs}}$  against  $2A_{\text{single}}$  (open blue circles) for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ ; (c) a histogram of the individual observed Flack (1983)  $x$  parameters for all data (the extreme histogram bins at  $-6$  and  $+6$  contain all data points for  $x < -6.0$  or  $x > 6.0$ , respectively); (d) like part (c), but for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ .

reflections are not used in this calculation.  $D_{\text{single}}$  is a model value for a single crystal (*i.e.* untwinned by inversion) using the atomic parameters determined as described in §2.1. The slope of the straight-line fit to  $D_{\text{obs}}$  against  $D_{\text{single}}$  is equal to  $1 - 2x$ , where  $x$  is the Flack (1983) parameter. So a slope of 1 corresponds to  $x = 0$ , a slope of  $-1$  to  $x = 1$  and a slope of zero to  $x = \frac{1}{2}$ . An absolute value of the slope greater than one does not have a physical interpretation. Two aspects of the calculation required experimentation. The first concerns the selection and rejection of data points and the second is the choice of weights.

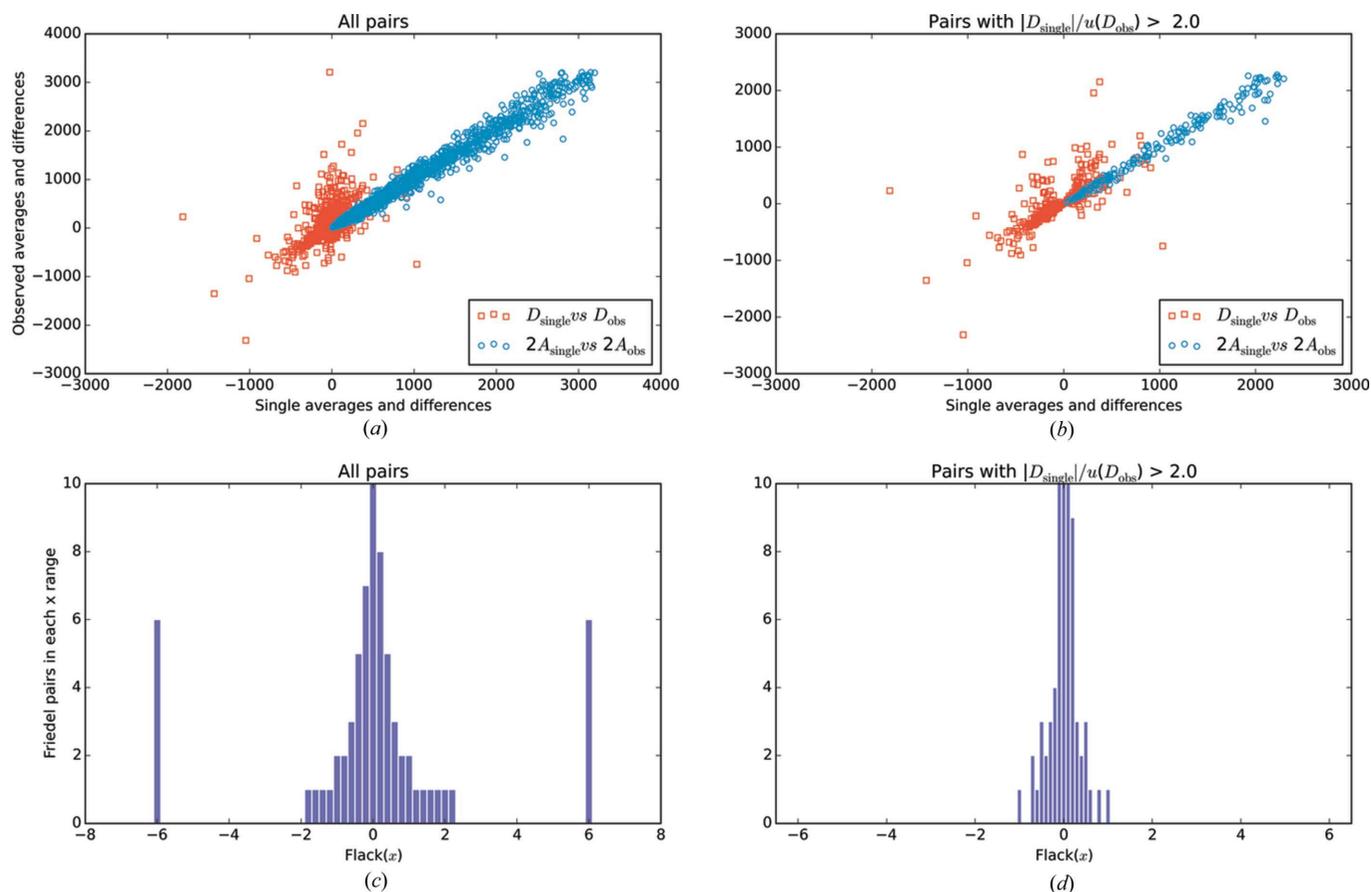
**2.2.1. Selection of data.** It is necessary to recall that  $D_{\text{obs}}$  values are always contaminated by random uncertainty and systematic error (Flack, 2014). For a good data set, the  $D_{\text{obs}}$  against  $D_{\text{single}}$  plot displays a scatter of points around a straight line passing through the origin. Data points whose  $D_{\text{obs}}$  values are dominated by random uncertainty and systematic error are distributed around the straight line  $D_{\text{single}} = 0$ , the value of  $|D_{\text{obs}}|$  often being much larger than the value of  $|D_{\text{single}}|$ . The resonant scattering signal in such data points is obscured by the experimental errors. As a result of these considerations, data were included in the post-refinement determination of

the Flack (1983) parameter by using the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > z$ , where in *CRYSTALS* the value  $z$  can be set in the ‘Absolute Structure Analysis’ tool in the box labelled ‘Filter(1)’. A value of  $z = 0.0001$  includes almost all the data whereas  $z = 0.5$  includes only the most significant. In the days of serial diffractometers, Le Page *et al.* (1990) used the value of  $|D_{\text{single}}|/u(D_{\text{obs}})$  as a criterion for the choice of those reflections most sensitive to the effects of resonant scattering.

**2.2.2. Weighting scheme.** Two systems of weights were the subject of study.

(i) The standard uncertainty  $u$  of  $D_{\text{obs}}$ , derived from the diffraction experiment and associated data reduction, was used to calculate weights as  $w(hkl) = 1/u^2[D_{\text{obs}}(hkl)]$ . If the errors in  $D_{\text{obs}}$  are random and adequately represented by  $u(D_{\text{obs}})$ , it is to be expected that this choice of weights would prove to be satisfactory.

(ii) The optimized weights of §2.1 for the refinement of atomic parameters were used for the determination of the Flack (1983) parameter. It was hardly expected that such weights would be satisfactory. The mean properties of  $A(hkl)$  (or  $|F(hkl)|^2$ ) and  $D(hkl)$  are quite different from one another as a function of  $\sin(\theta)/\lambda$  or other parameters. Moreover, their



**Figure 2**  
 Plots from wq3001 (Table 1), with  $x = 0.02$  (2), showing (a)  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data; (b)  $D_{\text{obs}}$  against  $D_{\text{single}}$  for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 2.0$ ; (c) a histogram of the individual observed Flack (1983)  $x$  parameters for all data; (d) like part (c), but for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 2.0$ .

dependence on atomic parameters and the Flack (1983) parameter are also strikingly different.

In *CRYSTALS*, there is a checkbox option to choose either the counting statistical weights,  $1/u_{\text{obs}}^2$ , or the weights as determined and applied in the main least-squares refinement.

**2.2.3. Results and evaluation.** Plots from bm3104III, with  $x = -0.01$  (4), are shown in Fig. 1. The data is of high quality and completeness, was measured at low temperature with Cu  $K\alpha$  radiation, the heaviest element in the material is oxygen and it has a  $\text{Friedif}_{\text{stat}}$  value of 22. Fig. 1(a) shows a plot of  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data. The  $D_{\text{obs}}$  values are distributed about the vertical line  $D_{\text{single}} = 0$ , a clear indication that many  $D_{\text{obs}}$  values are contaminated by random uncertainty and systematic errors. Fig. 1(b) shows a plot from the same data selecting those data points for which  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ . Although there is a wide scatter of points, it is evident that they follow a straight line of approximately unit slope passing through the origin, in agreement with the value of the Flack (1983) parameter found by a least-squares fit. Fig. 1(c) shows a histogram of individual observed Flack (1983)  $x$  parameters for all data.  $x_{\text{obs}}(hkl)$  (see Watkin, 2016) is defined as  $x_{\text{obs}}(hkl) = \frac{1}{2}[1 - D_{\text{obs}}(hkl)/D_{\text{single}}(hkl)]$ . The histograms peak close to the value of the Flack (1983) parameter obtained by a least-squares fit of  $D_{\text{obs}}$  to  $D_{\text{single}}$ . A notable feature of the histogram is the high frequency of  $x_{\text{obs}}$  values with  $|x_{\text{obs}}| > 5$ .

Upon applying the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ , as shown in Fig. 1(d), the extreme values with  $|x_{\text{obs}}| > 5$  are eliminated, demonstrating that they arise from data points with large  $|D_{\text{obs}}|$  and small  $|D_{\text{single}}|$  values.

Plots from wq3001, with  $x = 0.02$  (2), are shown in Fig. 2. The data is of high quality and completeness, was measured at low temperature with Mo  $K\alpha$  radiation, the heaviest element in the material is zinc and it has a  $\text{Friedif}_{\text{stat}}$  value of 558. Figs. 2(a), 2(b), 2(c) and 2(d) are similar to Figs. 1(a), 1(b), 1(c) and 1(d), respectively. In the plot shown in Fig. 2(a) of  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data (2089 Friedel pairs), it is evident that there are two classes of data points: (i) those distributed about  $D_{\text{single}} = 0$  and (ii) those distributed about a straight line passing through the origin of approximately unit slope. Applying the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > 2.0$  results in Fig. 2(b) (for 397 Friedel pairs), in which the noise along the  $D_{\text{obs}}$  axis is largely eliminated, leaving the straight line of data points of unit slope. Fig. 2(c) shows the histogram of  $x_{\text{obs}}$  values. As for bm3104III, there are a large number of  $x_{\text{obs}}$  values with  $|x| > 3$  which are eliminated on applying the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > 2.0$ .

Plots from fg3255III, with  $x = 0.58$  (2), are shown in Fig. 3. The data is of high quality and completeness, was measured at low temperature with Mo  $K\alpha$  radiation, the heaviest element in the material is chlorine and it has a  $\text{Friedif}_{\text{stat}}$  value of 113.

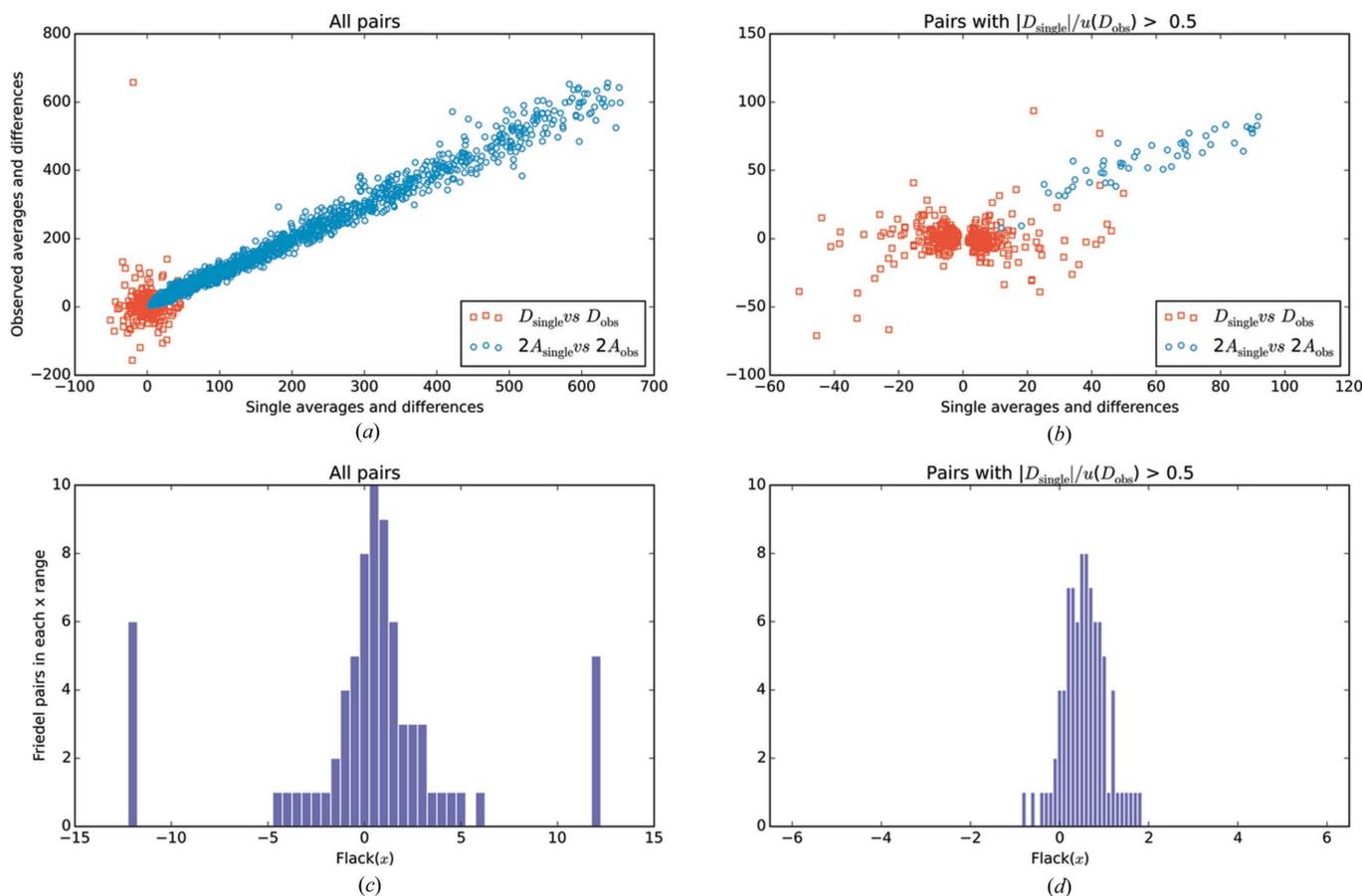


Figure 3

Plots from fg3255III (Table 1), with  $x = 0.58$  (2), showing (a)  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data; (b)  $D_{\text{obs}}$  against  $D_{\text{single}}$  for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ ; (c) a histogram of the individual observed Flack (1983)  $x$  parameters for all data; (d) like part (c), but for data with  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$ .

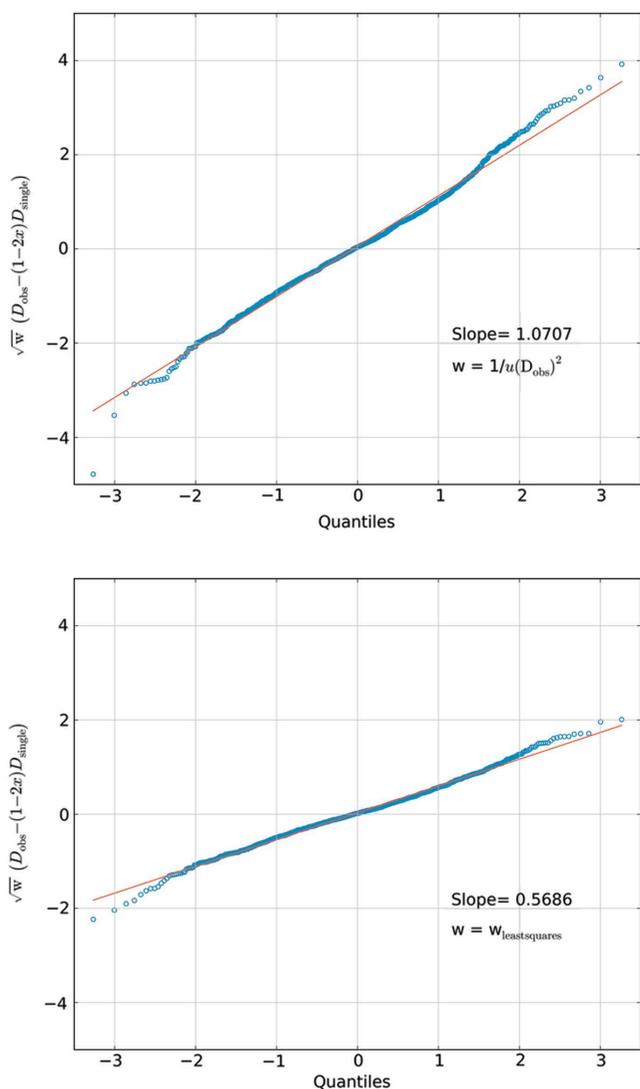
Figs. 3(a), 3(b), 3(c) and 3(d) are similar to Figs. 1(a), 1(b), 1(c) and 1(d), respectively. In the plot shown in Fig. 3(a) of  $D_{\text{obs}}$  against  $D_{\text{single}}$  for all data, the data points cluster around the origin with no apparent straight-line dependence. Applying the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$  results in Fig. 3(b) showing clearly the straight line of data points passing through the origin of slightly negative slope. The  $x_{\text{obs}}$  histogram in Fig. 3(c) shows that the highest frequency of  $x_{\text{obs}}$  values occur near  $x = 0.5$  and again there is a high frequency of  $x_{\text{obs}}$  values with  $|x| > 7$ . The latter are eliminated when the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > 0.5$  is applied.

It is very clear from a study of Figs. 1, 2 and 3 that the criterion  $|D_{\text{single}}|/u(D_{\text{obs}}) > z$  is a very powerful and useful tool for the selection and analysis of data for the determination of absolute structure. The value of  $z$  has to be chosen from a study of plots of the types presented in the figures. In *CRYSTALS*, the value of  $z$  may be set by the user. If the default value fails to yield sufficient reflections for the analysis, *CRYSTALS* suggests an alternative value.

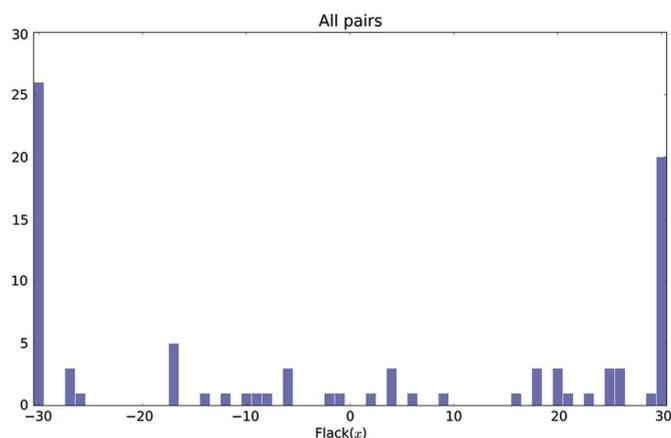
Normal probability plots of  $w^{1/2}[D_{\text{obs}} - (1 - 2x)D_{\text{single}}]$  from fn3089I, with  $x = 0.38$  (2), are shown in Fig. 4. The data is of high quality and completeness, was measured at low temperature with Mo  $K\alpha$  radiation, the heaviest element in the material is iron and it has a  $\text{Friedif}_{\text{stat}}$  value of 389. Figs. 4(a) and 4(b) show the normal probability plots for two different

weighting schemes. In Fig. 4(a), the weighting scheme is that based on the experimental standard uncertainties of the  $D_{\text{obs}}$  values, i.e.  $u(D_{\text{obs}})$ . In Fig. 4(b), the optimized weighting scheme is that used in the least-squares refinement of atomic parameters described in §2.1. The normal probability plot shows how closely the weighted errors follow a Gaussian distribution and deviations from a straight line with unit gradient highlight deviations from the expected distribution of errors. Fig. 4(b) shows that the weights used for least-squares refinement of all the parameters in the structure against all data are not appropriate for the post-refinement determination of the Flack parameter as they give rise to an error distribution which is too narrow, and hence a normal probability plot slope of approximately 0.5. Underestimation of weights arises from overestimation of the standard uncertainties associated with each observation. The normal crystallographic practice of augmenting standard uncertainties using a function of  $|F|$  and  $|F|^2$  in order to account for systematic error in the standard crystallographic model, or underestimated  $u(|F|^2)$  from an instrument, is not necessary in this case.

Referring to Table 1, one sees that in all cases but one (ov3013I), the standard uncertainty of the Flack (1983) parameter derived by the methods of §2.2 is, on average, a factor of 3.5 smaller than the value quoted by the original



**Figure 4**  
Normal probability plots of  $w^{\frac{1}{2}}[D_{\text{obs}} - (1 - 2x)D_{\text{single}}]$  from fn3089I (Table 1), with  $x = 0.38$  (2). In part (a), the weighting scheme is that based on the experimental standard uncertainties of the  $D_{\text{obs}}$  values, *i.e.*  $u(D_{\text{obs}})$ , and in part (b), the weighting scheme is the optimized one used in the least-squares refinement of the atomic parameters described in §2.1.



**Figure 5**  
Histogram of all  $x_{\text{obs}}$  for ov3013I (Table 1), with  $x = 5$  (17). The extreme histogram bins at  $-30$  and  $+30$  contain all data points for  $x \leq -30.0$  or  $x \geq 30.0$ , respectively.

authors (minimum and maximum values are 1.9 and 8.0, respectively). The value of the Flack (1983) parameter itself changes little between the two methods of determination. The only data set for which the standard uncertainty of the Flack (1983) parameter determined by the procedures described in §2.2 is larger than that found by the authors of the publication is ov3013I, *i.e.* the value of  $x$  is 6 (3) in ov3013I and 5 (17) in this work. A study of the histogram of all  $x_{\text{obs}}$  values in Fig. 5 helps to show how such a situation can arise. The histogram has the usual high frequency of  $x_{\text{obs}}$  values for  $|x| > 28$ ; however, no frequency peak is visible in the centre part of the histogram for  $|x| < 28$ . There is no preferred value of  $x$  and a very large standard uncertainty on  $x$  is a reasonable result in this case.

### 3. Concluding remarks

The results of this study are clear. Absolute structure determination may be carried out under the very best conditions in *CRYSTALS* using a variety of complementary tools for the analysis and selection of data. The tools permit visualization of the agreement between  $D_{\text{obs}}$  and  $D_{\text{single}}$ , and analysing the spread of  $x_{\text{obs}}$  values ensures the validity of the calculation. Moreover, it is not necessary to have the separate procedures *absolute-structure determination* and *no interest in absolute structure* as proposed in (Flack, 2014). One proceeds as follows:

(i) Using the complete set of diffraction intensities, merged and averaged in the point group of the space group, least-squares refinement is undertaken varying all general and atomic parameters defining the crystal structure. Initially, the Flack parameter can arbitrarily be set to zero. If, once the atomic model has stabilized, analysis of  $D_{\text{obs}}$  and  $D_{\text{single}}$  give a strong indication that the absolute structure of the major (or only) constituent of the sample is incorrect, the model should be inverted. If the analysis is ambiguous, the Flack (1983) parameter should be refined from a starting value of 0.5 simultaneously with the other parameters to ensure that the values of the atomic parameters (coordinates and atomic displacement parameters) are unbiased by the effect of resonant scattering. The least-squares weights may be adjusted to improve the fit, paying attention to obtain a uniform distribution of weighted residuals as functions of  $\sin(\theta)/\lambda$  and  $|F|^2$ . Plots of  $|F_{\text{obs}}|^2$  against  $|F_{\text{calc}}|^2$  and normal probability plots of  $w^{\frac{1}{2}}[|F_{\text{obs}}|^2 - |F_{\text{calc}}|^2]$  allow the outliers to be identified and the validity of the weights and standard uncertainties to be evaluated. The result of this part of the procedure is a set of unbiased atomic parameters with as small and realistic standard uncertainties as allowed by the data.

(ii) In the post-refinement step, the most reliable values of the Flack (1983) parameter and its standard uncertainty are calculated. For this purpose, only paired ( $hkl$  and  $\bar{h}\bar{k}\bar{l}$ ) acentric reflections are used and values of  $D_{\text{obs}}$  and  $D_{\text{single}}$  are calculated.  $D_{\text{obs}}$  comes directly from the observed intensities, whereas  $D_{\text{single}}$  is a calculated value for a single crystal (*i.e.* untwinned by inversion) of the model obtained in stage (i). The value of the Flack (1983) parameter is obtained from a least-squares fit of a straight line passing through the origin,

*i.e.* of  $D_{\text{obs}}$  against  $D_{\text{single}}$ . The slope of this line is equal to  $(1 - 2x)$ , where  $x$  is the Flack (1983) parameter. The  $D_{\text{obs}}$  and  $D_{\text{single}}$  data points to be retained in this procedure are those with  $|D_{\text{single}}|/u(D_{\text{obs}}) > z$ . The default value of 0.5 may usefully be decreased in the presence of strong resonant scatterers (a value of  $z = 0.0001$  includes almost all the data). The choice of an appropriate value of  $z$  is assisted by the graphical analyses presented. Smaller values of  $z$  allow more Friedel pairs into the calculation and small changes result in slightly lower values of  $u(x)$ , but very little change in  $x$ . The  $D_{\text{obs}}$  against  $D_{\text{single}}$  plots enable one to monitor the consequence of the inclusion of progressively less influential data as  $z$  is decreased. Further analysis of the effect of data selection is undertaken in Watkin (2016). Weights of  $1/u^2(D_{\text{obs}})$  produced the most satisfactory results and are chosen in *CRYSTALS* by selecting the ‘counting statistical weights’ checkbox. The fit of the data may be judged by the normal probability plot of  $[D_{\text{obs}} - (1 - 2x)D_{\text{single}}]/u(D_{\text{obs}})$ , by the histogram of Flack (1983)  $x$  parameters from individual  $D_{\text{obs}}$  values and by the  $D_{\text{obs}}$  against  $D_{\text{single}}$  scattergram. In the output of *CRYSTALS*, the value of the Flack (1983) parameter and its uncertainty determined by the methods described in this paper are identified by the label ‘Post-Refinement Flack’.

## References

- Bernardinelli, G. & Flack, H. D. (1985). *Acta Cryst.* **A41**, 500–511.
- Betteridge, P. W., Carruthers, J. R., Cooper, R. I., Prout, K. & Watkin, D. J. (2003). *J. Appl. Cryst.* **36**, 1487.
- Bojarska, J., Maniukiewicz, W., Sieron, L., Fruzinski, A., Kopczacki, P., Walczynski, K. & Remko, M. (2012). *Acta Cryst.* **C68**, o341–o343.
- Cora, I., Czugler, M., Dódony, I. & Rečnik, A. (2011). *Acta Cryst.* **C67**, i33–i35.
- Flack, H. D. (1983). *Acta Cryst.* **A39**, 876–881.
- Flack, H. D. (2008). *Acta Chim. Slov.* **55**, 689–691.
- Flack, H. D. (2013). *Acta Cryst.* **C69**, 803–807.
- Flack, H. D. (2014). *Chimia*, **68**, 26–30.
- Flack, H. D. & Bernardinelli, G. (1999). *Acta Cryst.* **A55**, 908–915.
- Flack, H. D. & Bernardinelli, G. (2000). *J. Appl. Cryst.* **33**, 1143–1148.
- Flack, H. D. & Bernardinelli, G. (2008). *Chirality*, **20**, 681–690.
- Flack, H. D., Sadki, M., Thompson, A. L. & Watkin, D. J. (2011). *Acta Cryst.* **A67**, 21–34.
- Frampton, C. S., MacNicol, D. D. & Wilson, D. R. (2011). *Acta Cryst.* **C67**, o188–o191.
- Hendsbee, A. D., Masuda, J. D. & Piórko, A. (2011). *Acta Cryst.* **C67**, m391–m394.
- Hooft, R. W. W., Straver, L. H. & Spek, A. L. (2008). *J. Appl. Cryst.* **41**, 96–103.
- Hooft, R. W. W., Straver, L. H. & Spek, A. L. (2010). *J. Appl. Cryst.* **43**, 665–668.
- Kefi, R., Jeanneau, E., Lefebvre, F. & Ben Nasr, C. (2011). *Acta Cryst.* **C67**, m126–m129.
- Le Page, Y., Gabe, E. J. & Gainsford, G. J. (1990). *J. Appl. Cryst.* **23**, 406–411.
- Li, Y., Liu, Q.-K., Ma, J.-P. & Dong, Y.-B. (2012). *Acta Cryst.* **C68**, m152–m155.
- Ojala, W. H., Arola, T. M., Brigino, A. M., Leavell, J. D. & Ojala, C. R. (2012). *Acta Cryst.* **C68**, o270–o278.
- Parsons, S., Flack, H. D. & Wagner, T. (2013). *Acta Cryst.* **B69**, 249–259.
- Parsons, S., Pattison, P. & Flack, H. D. (2012). *Acta Cryst.* **A68**, 736–749.
- Ślepokura, K. (2012). *Acta Cryst.* **C68**, o311–o316.
- Thompson, A. L. & Watkin, D. J. (2011). *J. Appl. Cryst.* **44**, 1017–1022.
- Watkin, D. J. (2016). In preparation.
- Wölper, C., Anwar, N., Gulzar, N., Jones, P. G. & Blaschette, A. (2011). *Acta Cryst.* **C67**, o249–o254.
- Zhong, K.-L. & Qian, M.-Y. (2012). *Acta Cryst.* **C68**, m265–m268.